Modeling of hysteresis by means of a directional approach. Constitutive Models

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Modeling of hysteresis by means of a directional approach

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ABSTRACT: This paper focuses on the mechanical hysteresis in elastomers, i.e. the difference between loading and unloading paths. This property can be time-dependent as well as time-independent, depending on the physical phenomena that come into play. Similarly, mechanical hysteresis can be affected or not by material anisotropy. In this context, the present study is devoted to the modeling of time-independent hysteresis, in the framework of material anisotropy, accommodated to the Mullins effect. For this purpose, directional model is used to predict the tridimensional response of such materials. The proposed model is based on the stress decomposition into two parts. The first one represents the hyperelasticity of the macromolecular network, whereas the second part represents the friction in the network, i.e. the hysteretic part. Experiments were carried with filled silicone rubber and results show that the model predictions and experimental curves fit well.

1 INTRODUCTION

The characterization and modeling of rubber-like materials behavior have been widely addressed in last decades due to the increasing number of industrial applications. In order to improve the physical properties of the material, silicone rubbers are classically filled by mineral fillers, implying an increase of stiffness but also the appearance of some unconventional effects. Among these phenomena, one can cite the stress softening, which principaly occurs between first and second loads, often called the Mullins effect (Mullins, 1948), the relaxation and the hysteretic behavior (unload different from load), non-exhaustively.

Moreover, the load and unload responses of a rubber-like material differ during cyclic tests and form a mechanical hysteresis. This phenomenon depends on the strain rate (Amin *et al.*, 2006), the crosslinks density (Bergström and Boyce, 1998) and the temperature (Rey *et al.*, 2013b).

Several micro-motivated (Bergström and Boyce, 1998; Miehe and Göktepe, 2005; D'Ambrosio *et al.*, 2008; Lorenz and Klüppel, 2012) and phenomenological (Dorfmann and Ogden, 2003) approaches of the mechanical hysteresis were published in the literature.

Despite the large number of papers focusing on the viscoelastic behavior of the material, very few studied the rate-independent hysteresis (Kaliske and Rothert, 1998; Vandenbroucke *et al.*, 2010). In order to distinguish stress-softening and hysteresis, very few authors remove the Mullins effect from the material by carrying out several previous mechanical cycles (Lion, 1996; Bergström and Boyce, 1998).

This paper focuses on the quasi-static modeling (*i.e.* the rate independent behavior) of this hysteresis phenomenon after stress softening. The aim of this work is to propose a directional method able to predict the difference between load and unload curves, usable with existing models, and easily extendable to anisotropy. The proposed theory to model hysteresis is introduced in part 2. In part 3, the experimental data are compared with simulations and these results are discussed. Finally, some concluding remarks close the paper.

2 THEORY

2.1 *Material behavior*

Classically, models are based on a decomposition of the material behavior into several contributions related to different physical phenomena. As an example, a first hyperelastic part can be coupled to a viscous part, a damage part or both of them. As the viscosity is not taken into account in this paper, the model used contains only a hyperelastic part, represented by a non linear spring, and a hysteretic part, represented by an infinity of spring-friction slider associations. Even if this kind of representation is used for the elastomers, it comes from the modeling of other materials behavior, as that of for metallic alloys (Guélin, 1980; Favier and Guélin, 1985).

The mechanical response of the material is, in this paper, obtained by adding the two previous parts, as previously done in Favier (1988); Favier *et al.* (1992) for shape memory alloys and magnetisation phenomena. The total response, with the incompressibility hypothesis, is defined as:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_{rev} + \boldsymbol{\sigma}_{hyst} - p\mathbf{I} \tag{1}$$

where p is the hydrostatic pressure.

2.2 *Hyperelastic part*

Any hyperelastic potential can be used. In this paper, the model proposed by Biderman (1958) is chosen:

$$\begin{aligned} W_{rev} &= c_{10}(I_1 - 3) + c_{20}(I_1 - 3)^2 \\ &+ c_{30}(I_1 - 3)^3 + c_{01}(I_2 - 3) \end{aligned} \tag{2}$$

where c_{10}, c_{20}, c_{30} and c_{01} are material parameters, I_1 and I_2 are respectively the first and second invariants of the right Cauchy-Green tensor defined as $I_1 = tr(\mathbf{C})$ and $I_2 = \frac{1}{2}(tr(\mathbf{C})^2 - tr(\mathbf{C^2}))$.

2.3 *Hysteresis part*

2.3.1 *General form*

A model with an infinity of directions in the space is used. In the case of an initially isotropic material, chains are assumed to be equidistributed in space. This approach was previously used in the literature (Wu and Van der Giessen, 1993). However, to avoid integration problems, a discrete number of directions is used. Here, the 42 directions representation of Bažant and Oh (1986), even if other special repartitions can be chosen. This representation was already used to model rubber-like materials, particularly Mullins effect, see for example Itskov *et al.* (2006), Diani *et al.* (2006) and Rebouah *et al.* (2013). Initial directional vectors $\mathbf{a_0}^{(i)}$ are defined, and initial direction tensors $\mathbf{A_0}^{(i)}$, current direction vectors $\mathbf{a}^{(i)}$ and tensors $\mathbf{A}^{(i)}$ are respectively calculated by:

$$\begin{aligned} \mathbf{A_0}^{(i)} &= \mathbf{a_0}^{(i)} \otimes \mathbf{a_0}^{(i)} \\ \mathbf{a}^{(i)} &= \mathbf{F}\mathbf{a_0}^{(i)} \\ \mathbf{A}^{(i)} &= \mathbf{a}^{(i)} \otimes \mathbf{a}^{(i)} \end{aligned} \tag{3}$$

where $\mathbf{F}$ is the deformation gradient.

By means of the spatial repartition, only monodimensional constitutive equations are needed to represent a tridimensional behavior. No tridimensional generalization of monodimensional constitutive equations (Favier *et al.*, 1992) are needed as in Laurent *et al.* (2008) and Vandenbroucke *et al.* (2010).

In each direction *(i)* the considered constitutive equation during the first load is expressed as:

$$\sigma^{(i)} = S_0 \tanh \frac{E \ln I_4^{(i)}}{2\sigma_0} \tag{4}$$

where $\sigma^{(i)}$ is the Cauchy stress in the considered direction, E is equivalent to an initial slope, σ_0 is the maximum reachable Cauchy stress by the hysteresis part on each direction $a_0{}^{(i)}$, and $I_4^{(i)}$ is the fourth invariant of the right Cauchy-Green tensor ($\mathbf{C} = \mathbf{F^T F}$) defined as $I_4^{(i)} = tr(\mathbf{C A_0}^{(i)})$, where tr means the trace. The chains are assumed to bring a contribution to the material behavior only in the case of a tension loading. Thus, they have no significant contribution in compression. The stress tensor is finally obtained by summing the contributions of each direction:

$$\boldsymbol{\sigma}_{hyst} = \sum_{i}^{42} \omega_i \sigma^{(i)} \left(\mathbf{A}^{(i)} - \frac{I_4^{(i)}}{3}\mathbf{I} \right) \tag{5}$$

where ω_i are the weight corresponding to the considered direction, defined by Bažant and Oh (1986), and $\mathbf{I}$ the identity tensor.

2.3.2 *Inversion point*

Inversion points are introduced to create the difference between loads and unloads (Guélin, 1980; Wack *et al.*, 1983). At the end of a load, the constitutive equation in a direction (i) during unloading is expressed as:

$$\sigma^{(i)} = \sigma_{inv}^{(i)} - \sigma_0 \tanh \frac{E(\ln I_{4inv}^{(i)} - \ln I_4^{(i)})}{2\sigma_0} \tag{6}$$

where $\sigma_{inv}^{(i)}$ and $\ln I_{4inv}^{(i)}$ are respectively the stress and strain of the last inversion point. It is to note that this equation is the same as Eq. (4) except the change in the origin function. The readers can refer to Rey *et al.* (2013a) for further informations.

3 EXPERIMENTAL VALIDATION

3.1 *Material*

The rubber-like material used is a filled silicone (Bluestar RTV 3428). The mechanical properties of this material were investigated by Machado *et al.* (2010, 2012) and Rey *et al.* (2013b).

3.2 *Experimental devices and specimen preparations*

Different uniaxial tensile tests are carried out in order to validate the model. These tests are performed by means of the Gabo Eplexor 500N, whose load cell capacity is 25N. The length, width and thickness of the uniaxial tensile tests specimens are 12, 2 and 2 mm respectively.

Preconditioning tests are carried out on the specimens to remove the Mullins effect. This enables us to ensure the repeatability of the tests (Lion, 1996; Bergström and Boyce, 1998; Dorfmann and Ogden, 2004). This preconditioning procedure consists in performing 5 cycles of load-unload at a maximum stretch of 3, with an strain rate equal to $1.67s^{-1}$.

First investigations carried out by Rey *et al.* (2013a) demonstrate that strain rate inferior to $1.67s^{-1}$ does not influence the mechanical behavior.

3.3 *Loading conditions*

Different uniaxial tensile tests are carried out. A first cyclic test is performed from 1.25 to 2.5 stretch level, with an increase of 0.25 between each cycle.

Two additional uniaxial tensile tests are performed to characterize the hysteresis phenomenon. The first test (called nc1 in the following) is a load-unload cycle with two hysteresis loops during the load and the unload. The second (nc2) is a load-unload cycle with a hysteresis subloop inside a first loop during the load.

4 MODEL VALIDATION

The constitutive parameters are fitted from experimental data obtained during the first test. They are reported in table 1.

$c_{10}(MPa)$	$c_{20}(MPa)$	$c_{30}(MPa)$
0.095	-0.0015	0.00059
$c_{01}(MPa)$	$E(MPa)$	$S_0(MPa)$
0.03	0.9409	0.06

Table 1: Values of the material parameters fitted on experimental data.

The simulation of the first uniaxial tensile test with increasing cycles from 1.25 to 2.5 stretch level is shown in figure 1, where the nominal stress (or Piola-Kirchhoff 1 stress, *i.e.* the current force divided by the initial section) versus the stretch λ (*i.e.* the ratio between current and initial length) is plotted. It can be seen in this figure that the silicone behavior is well predicted by the model for this experiment as the two curves are nearly superposed. The results of the simulation of 'nc1' is presented in figure 2. In this case, the model gives a reasonable prediction of the stress response of the behavior. Figure 3 shows the results of the simulation of the test 'nc2'. The curves are nearly counfounded, meaning that the model gives good predictions of the material behavior in this case.

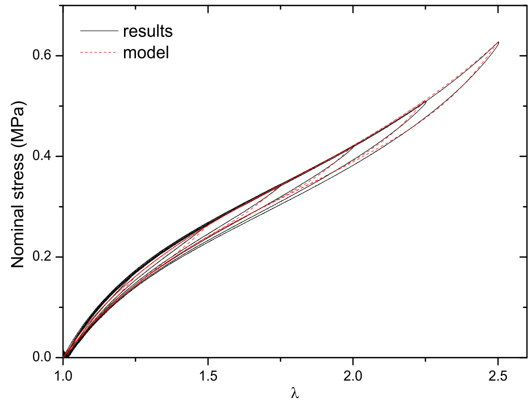

Figure 1: Simulation of a uniaxial tensile test with increasing cycle.

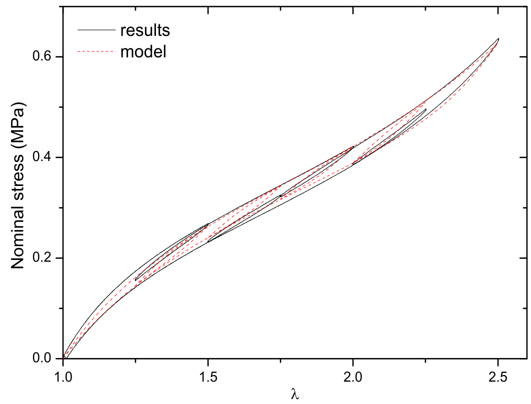

Figure 2: Simulation of a uniaxial tensile test with four hysteresis loops (nc1 test).

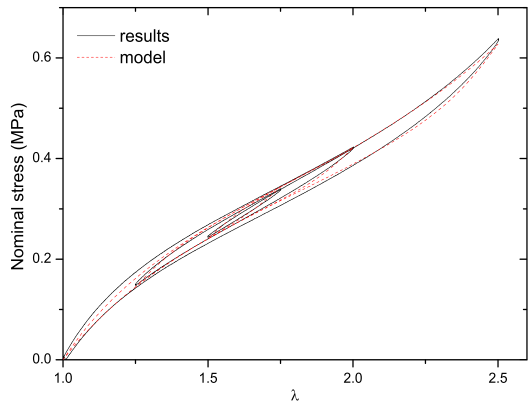

Figure 3: Simulation of a uniaxial tensile test with a hysteresis loop inside another one (nc2 test).

5 CONCLUSION

A directional elasto-hysteresis model for rubberlike materials was developed in this study. Different uniaxial tests were carried out with hysteresis loops. The proposed model predicts well the material behavior during these different tests.

Further investigations, like other tests and numerical implementation, are carried out by the authors in Rey *et al.* (2013a).

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